LS-KS(7-24-87)

LS-99
July 1987

MEAN TEMPERATURE RISE IN A TARGET

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The equation which determines the equilibrium temperature distribution in a cylindrically symmetrical target, if we deposit an average power J(r) inside radius r, is

$$J(r) = -2\pi r \ell \kappa \frac{dT}{dr}, \tag{1}$$

where κ is the thermal conductivity and ℓ is the length of the target. The temperature is then

$$T = T_o - \frac{1}{2\pi\kappa\ell} \int_o^r \frac{J(r)dr}{r}. \tag{2}$$

If we deposit power uniformly in a cylinder of radius a, then

$$J(r) = \begin{cases} J_o r^2/a^2 & , r \leqslant a, \\ J_o & , r \geqslant a, \end{cases} \tag{3}$$

where J_o is the total power delivered. The temperature is then

$$T = \begin{cases} T_o - \dfrac{J_o r^2}{4\pi\kappa\ell a^2}, & r \leqslant a, \\ T_o - \dfrac{J_o}{4\pi\kappa\ell} (1 + 2 \ \ell n \ r/a), & r \geqslant a. \end{cases} \tag{4}$$

If the target boundary at r = b is held at a temperature T_b, then the temperature rise at the center is

$$\Delta T = T_o - T_b = \frac{J_o}{4\pi\kappa\ell}\,(1 + 2\,\ell n\, b/a). \tag{5}$$

The power is not deposited uniformly through the target. If the length ℓ of the target is 2 radiation lengths, then the power reaching a depth s is

$$J_s = J_o\, e^{-2s/\ell}, \tag{6}$$

and the power per unit length is

$$-\frac{dJ}{ds} = \frac{2J_o}{\ell}\, e^{-2s/\ell}. \tag{7}$$

The maximum temperature rise occurs at the front of the target and is

$$\Delta T = \frac{2J_o}{4\pi\kappa\ell}\,(1 + 2\,\ell n\, b/a). \tag{8}$$

This formula assumes that the radiation length is large compared with the radius of the beam, which is hardly true in the present case. Formula (8) is then an over-estimate, but should be good enough for our purposes. Moreover, not all the electron energy is deposited in the target. Some of the radiation produced is deposited later in the target, and some escapes. The maximum energy deposition may not occur at the target entrance, and will be less than predicted by formula (8), which is therefore conservative.

The thermal conductivity of tungsten at room temperature is 0.5 cal/sec cm degC, and about half that at 2000°C. The conceptual design value for the total electron power is 80 W. If we put b = 2a, ℓ = 0.7 cm, formula (8) gives a temperature rise of about 22°, if we use the room temperature value of the conductivity. The actual rise will be even less, since not all the power is deposited in the target.

We conclude that the mean temperature rise in the target should not be a problem. It remains to discuss the effect of the thermal shock due to the sudden deposit of an electron pulse in a small region in the target. A rule of thumb among target designers [1] is that if we exceed 200 joules per gram deposited suddenly in tungsten, corresponding to a 1500 degree temperature rise, the resulting shock will destroy the target. At a 1-Hz repetition rate, the beam delivers 80 J or 20 calories to the target per second. If we assume this is all deposited suddenly and uniformly in a cylinder of 3-mm diameter, 7-mm long, and take the specific heat of tungsten as 6 cal/mole °C, and the density 0.1 mole/cc, we get an instantaneous temperature rise of 700°. If we double this to take account of the non-uniform energy deposition, we get for the instantaneous temperature rise at the front face: 1400°. This would be close to the limit, but safe, since not all the energy is deposited in the target. In the actual design, the energy is deposited in eight pulses spaced 17 ms apart. Since the speed of sound in tungsten is about 4000 mm/ms, and the target region is only a few mm in size, each pulse produces a separate shock, with a temperature rise less than 1400/8 or 175 degrees, so the conceptual design is perfectly safe.

The new design proposes to deliver one half the charge of the conceptual design in eight pulses over 1.2 microsec. The shock will travel a few mm in that time; however, to be conservative, let us treat all eight pulses as one instantaneous thermal pulse. That is 40 J delivered to the target, which gives a temperature rise less than 700 degrees, which is safe. There are fifteen pulses per second, so that formula (8) gives a mean temperature rise of 165 degrees, which is also tolerable. Note that if the synchrotron were to run at one hertz, and if the total 600 J per second were delivered to the target in one pulse, then the instantaneous temperature rise would be of the order of 10,000 degrees, which would be disastrous.

REFERENCE

[1] F. E. Mills, private communication.